Mathematical Modeling of Stress Relaxation Non-Newtonian Blood Flow through an Artery with Multiple Stenosis.

Abstract: In this theoretical study a mathematical model is developed for studying stress relaxation property of blood flow through a narrow artery with multiple stenoses. The analytical results for velocity and flux are derived using appropriate boundary conditions. The derived quantitative analysis is performed for the flow velocity, flux, the resistive impedances and the wall shear stress with their variation with time are discussed. The axial velocity as a function for different values of Jeffrey parameter in the stenotic region is presented graphically.

Keywords: stress relaxation, narrow artery, multiple stenoses, flux, wall shear stress, axial velocity, Jeffrey parameter, stenotic region.

Introduction: Stenosis means narrowing arteries. This abnormality in arteries is perturb the flow field. The stenosed artery having 50%-90% area reduction was considered. Mathematical modeling and analysis of an artery with multiple stenosis is of great use for medical Scientists and Bio-mathematicians because it involves the investigation of various characteristics of blood. The objectives of this study is to investigate the flow mechanics of Non-Newtonian blood through an artery with multiple stenosis which will be beneficial in the field of medical sciences as the medical Scientists do not have precise information about various flow parameters of blood.

There is strong evidence that hydrodynamic factors such as resistance to flow, wall shear stress and apparent viscosity may play a vital role in the development and the progression of arterial stenosis. A number of studies related to blood flow through stenosed arteries have been carried out. Most of the studies have focused on Non-Newtonian blood flow. Mathur and Jain (2013) studied Mathematical modeling of Non-Newtonian blood flow through artery in the presence of stenosis. They have dealt with the effects of stenosis in an artery by considering the blood as power-law fluid. They have been concluded that the pressure drop and shear stress increases as the size of the stenosis increases for a given non-Newtonian model of the blood. Ramesh Babu and Savita (2019) investigated flow of Jeffrey fluid through an artery with multiple stenosis to examine the variation of velocity profiles and volumetric flow rate in different regions of flow with some boundary conditions. Halder et. al. (2017) studied Newtonian and Non-Newtonian pulsatile flows through an artery with stenosis and presented three-dimensional modeling and analysis of blood flow through artery stenosis under several variants of pulsatile flow to mimic the atherosclerosis artery disease. Sriyab (2020) analyzed the effect of stenotic geometry and Non-Newtonian property of blood flow through arterial stenosis. They considered a mathematical model of non-Newtonian blood flow through different stenosis, namely bell shape and cosine shape. He also concluded that the stenosed artery geometry, the stenosis length, stenosis depth and the power law index (non-Newtonian behavior) are important factors affecting the blood flow through the stenosed artery. Shit et. al. (2012) studied Mathematical modeling of blood flow through a tapered overlapping stenosed artery with variable viscosity. They observed that the influence of hematocrit, magnetic field and the shape of artery have important impact on the velocity profile, pressure gradient and wall shear stress. In their study the variable viscosity of blood depending on hematocrit and the blood has been treated as the porous medium. The problem was solved analytically by using the Frobenius method. Nanda and Bose investigated a mathematical model for blood flow through a narrow artery with multiple stenoses. In their investigation they discussed the rheological parameters, height of stenosis and yield stress of the fluid is strongly influence the flow characteristics qualitatively and quantitatively. Nanda and Mallik (2012) analyzed a Non-Newtonian two phase fluid model for blood flow through arteries under stenotic condition. Their analysis was carried out performing large scale numerical of the measurable flow variables having more physiological significance by developing computer codes. Chakravarty and Mandal (2000) studied two-dimensional blood flow through tapered arteries under stenotic conditions. In this analytical treatment bears the potential to calculate both the axial and the radial velocity profiles with low computational complexity by exploiting the appropriate boundary conditions and the input pressure gradient arising from the normal functioning of the heart.

The present study is motivated towards a theoretical investigation of mathematical modeling of stress relaxation Non-Newtonian blood flow through an artery with multiple stenosis. The analytical results for flow velocity and flux are derived. The derived analytical expressions are enumerated in order to examine the variation of velocity profiles and volumetric flow rate in different regions of flow.

ISSN 2229-5518

MATHEMATICAL FORMULATION:

Consider the steady flow of a Jeffrey fluid through a tube with non-uniform crosssection and with two stenoses. I consider the cylindrical polar co-ordinate system (r, z) so that z measured along the tube axis and r normal to the axis of the tube.. The stenoses are mild and axially symmetric.

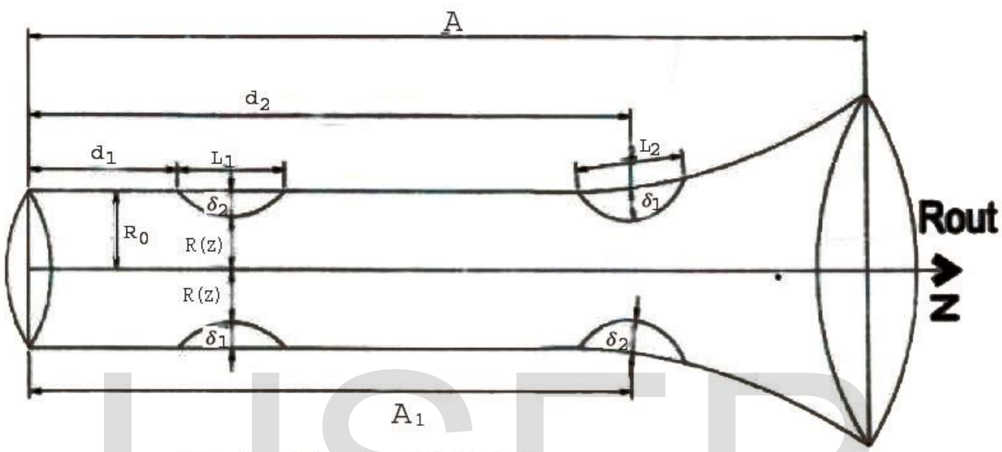

Fig 1 : Physical Model

The radius of the tube is taken as h(z) = R(z) and

$$R(z) = R_0 \qquad \text{where } 0 \le z \le d_1$$

$$= R_0 - \frac{\delta_1}{2}\left\{1+ \cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right\} \qquad \text{where } d_1 \le z \le d_1 + L_1$$

$$= R_0 \qquad \text{where } d_1 + L_1 \le z \le A_1 - \frac{L_1}{2}$$

$$= R_0 - \frac{\delta_2}{2}\left\{1+ \cos\frac{2\pi}{L_2}\left(z - A_1 - \frac{L_2}{2}\right)\right\} \qquad \text{where } A_1 - L_2 \le z \le A$$

$$= R^*(z) - \frac{\delta_2}{2}\left\{1+ \cos\frac{2\pi}{L_2}\left(z - A_1 - \frac{L_2}{2}\right)\right\} \qquad \text{where } A_1 \le z \le A_1 + \frac{L_2}{2}$$

$$= R^*(z) \qquad \text{where } A_1 + \frac{L_2}{2} \le z \le A \quad \ldots\ldots\ldots\ldots (1)$$

Here, lengths of the two stenoses are L_i (where i = 1, 2) and maximum thickness of two stenoses are δ_i (where i =1, 2) and the restrictions for the mild stenosis are satisfied.

$$\delta_i << \min (R_0, R_{out})$$

ISSN 2229-5518

$$\delta_i << L_i \ (i = 1, 2)$$

Where $R_{out} = R(z)$ at z = A.

The basic equation managing the flow is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\mu_1}\frac{\partial w}{\partial r}\right) = \frac{1}{\rho}\frac{\partial p}{\partial z} \ \ldots\ldots\ldots\ldots.. (2)$$

Where μ_1 is Jeffrey parameter, p is the pressure, ρ is the viscosity of the fluid, R_0 is the radious of the tube.

The boundary conditions are

$$\frac{\partial w}{\partial r} = 0 \quad \text{when } r = 0 \ \ldots\ldots\ldots\ldots(3)$$

$$w = 0 \quad \text{when } r = R(z) \ \ldots\ldots\ldots\ldots(4)$$

Introducing following non-dimensional variables

$$\bar{r} = \frac{r}{R_0} \quad \bar{p} = \frac{pR_0{}^2}{\rho UA} \quad \bar{d} = \frac{d_1}{A}$$

$$\overline{L_2} = \frac{L_2}{A} \quad \bar{w} = \frac{w}{\rho} \quad \bar{z} = \frac{z}{A}$$

$$\overline{L_1} = \frac{L_1}{A} \quad \overline{A_1} = \frac{A_1}{A} \quad \bar{R}(z) = \frac{R(z)}{R_0}$$

$$\bar{Q} = \frac{Q}{\pi U R_0{}^2} \quad \bar{\delta_\iota} = \frac{R(z)}{R_0} \ \ldots\ldots\ldots\ldots (5)$$

Non-dimensionalising the governing equations after dropping bars

$$\frac{\partial}{\partial r}\left(\frac{r}{1+\mu_1}\frac{\partial w}{\partial r}\right) = r\frac{\partial p}{\partial z} \ \ldots\ldots\ldots\ldots (6)$$

The non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r} = 0 \quad \text{when } r = 0 \ \ldots\ldots\ldots\ldots(7)$$

$$w = 0 \quad \text{when } r = R(z) \ \ldots\ldots\ldots(8)$$

Solution of the problem

(i) Velocity Distribution

Integrating eqn (1) by applying boundary conditions (7) and (8) the axial velocity can be obtained as

$w = \frac{1}{4(1+\mu_1)} \frac{\partial p}{\partial z} (R^2 - r^2)$ ……….. (9)

The volumetric rate of flow is obtained as

$Q = \frac{1}{16(1+\mu_1)} \frac{\partial p}{\partial z} R^4(z)$ ……….. (10)

(ii) Pressure Difference

The pressure difference Δp along the total length of the tube as follows

$\Delta p = \int_0^1 \frac{Q16(1+\mu_1)}{R^4(z)}$ …………. (11)

Result and Discussion:

I have calculated the axial velocity as a function of r for different values of Jeffrey parameter μ_1 from equation (9) in the stenotic regions $d_1 \leq z \leq d_1 + L_1$ and $A_1 - \frac{L_2}{2} \leq z \leq A_1$ and is shown in the figures (2) and (3). It is observed that the velocity decreases with the increase in the Jeffery parameter μ_1 in both the stenotic regoins.

The volumetric flux is calculated from equation (10) for different Jeffery parameter μ_1 in the stenotic region $d_1 \leq z \leq d_1 + L_1$ and is shown in the figure (4). The curve resemble an inverted parabola. It is noticed that minimum flux rate is attained at z = 0.3, that is the mid point of the stenosis.

Corresponding to the stenotic region $A_1 - \frac{L_2}{2} \leq z \leq A_1$ the volumetric flux is calculated for different Jeffery parameter and is shown in the figure (5). It is observed that the flux rate decreases with increase in Jeffery parameter μ_1 in the stenotic region.

The volumetric flux is calculated for different values of k and is shown in figure (6). In the third stenotic region for numerical computation I assume that

$$\frac{R^*(z)}{R_0} = e^{k(z-A_1)^2} = e^{\bar{k}(\bar{z}-\overline{A_1})^2} \text{ where } \bar{k} = k A^2$$

And $\overline{d_1} = 0.2$, $\overline{L_1} = 0.2$, $\overline{A_1} = 0.2$, $\overline{L_2} = 0.8$.

Here I have seen that flux increases in the value of k. The varaiation of Q the flow rate for different values of Jeffery parameter μ_1 in the third stenotic region $A_1 \leq z \leq A_1 + \frac{L_2}{2}$ is shown in the figure (7). Here it is seen that flux decreases as Jeffery parameter μ_1 increases.

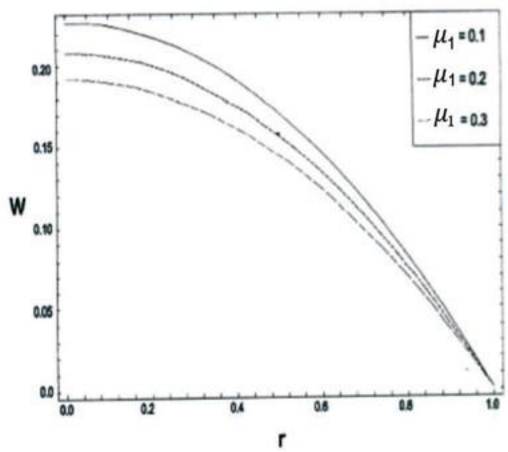

Fig 2: Velocity profiles for different Jeffrey Parameter μ_1 in the first stenotic region

$d_1 \leq z \leq d_1 + L_1$

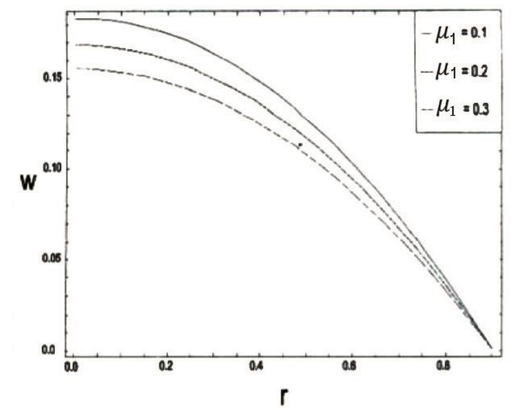

Fig 3: Velocity profiles for different Jeffrey Parameter μ_1 in the second stenotic region

$A_1 - \frac{L_2}{2} \leq z \leq A_1$

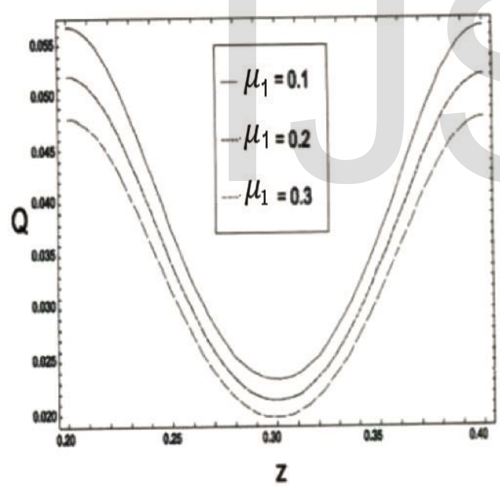

Fig 4: Volumeteric Flux for different Jeffrey parameter μ_1 in the first stenotic region

$d_1 \leq z \leq d_1 + L_1$

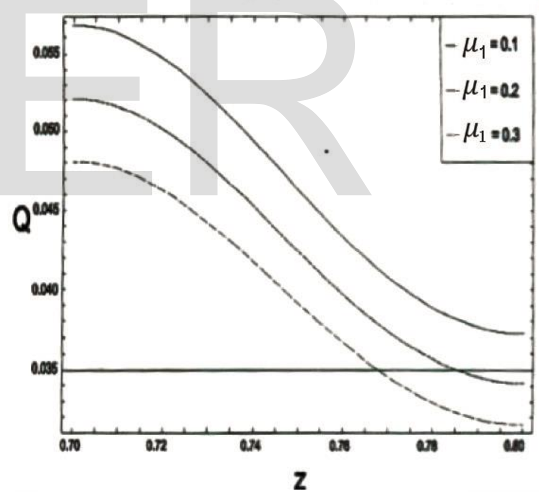

Fig 5: Volumeteric Flux for different Jeffrey parameter μ_1 in the second stenotic region

$A_1 - \frac{L_2}{2} \leq z \leq A_1$

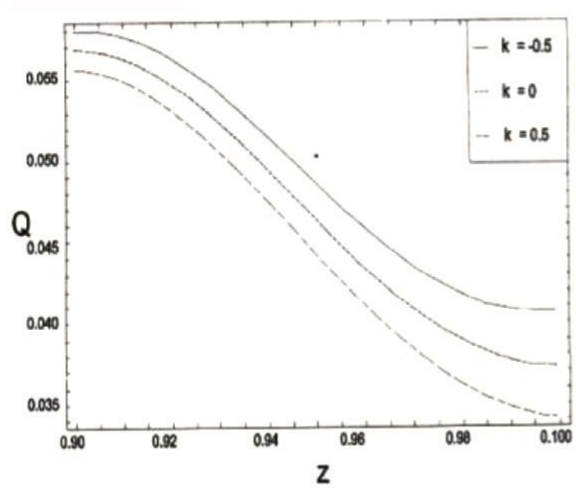

Fig 6: Volumetric Flux for different k in the third stenotic region

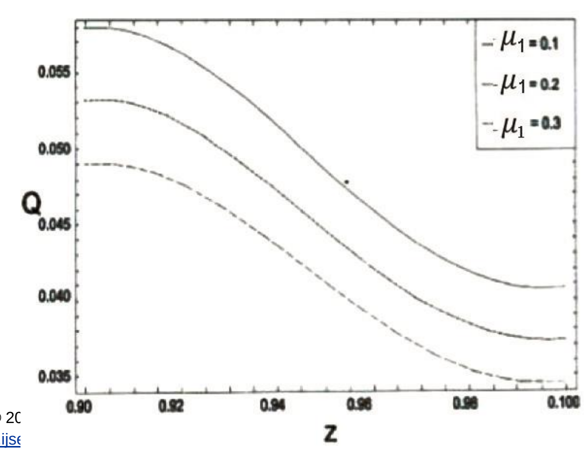

Fig 7 : The Volumetric Flux for diffenent Jeffrey parameter μ_1 in the third stenotic region $A_1 \leq z \leq A_1 + \frac{L_2}{2}$

References:

1. Dash, R.K.; Jayaraman, G. and Mehta, K.N. (1999), "Flow in a catheterized curved artery with stenosis", Journal of Biomechanics, vol. 32, pp. 49–61.
2. Chakravarty, S. and Mandal, P.K. (2000), "Two-dimensional blood flow through tapered arteries under stenotic conditions" International Journal of Non-Linear Mechanics, vol. 35, pp. 779-793.
3. Srivastava, V. P. (2003), "Flow of a Couple Stress Fluid Representing Blood through Stenotic Vessels with a Peripheral Layer", Indian Journal of Pure and Applied Mathematics, vol. 34, pp. 1727-1740.
4. Pralhad, R.N. and Schultz, D.H. (2004), "Modeling of arterial stenosis and its applications to blood diseases", Mathematical Biosciences, vol. 190, pp. 203-220.
5. Misra, J.C. and Shit, G.C. (2006), "Blood flow through arteries in a pathological state: A theoretical study", International Journal of Engineering Science, vol. 44, pp. 662-671.
6. Ponalagusamy, R. (2007), "Blood Flow through an Artery with Mild Stenosis: A two – layered model, different shapes of stenoses and slip velocity at the wall", Journal of Applied Sciences, vol. 7, pp. 1071-1077.
7. Joshi, P.; Pathak, A. and Joshi, B.K. (2009), "Two-layered model of blood flow through composite stenosed artery", Applications and Applied Mathematics, vol. 4, pp. 343-354.
8. Srivastava, V. P. and Rastogi, R. (2009), "Effects of hematocrit on impedance and shear stress during stenosed artery catheterization", Applications and Applied Mathematics, vol. 4, pp. 98-113.
9. Singh, B.; Joshi, P. and Joshi, B.K. (2010) "Blood flow through an artery having radially non-symmetric mild stenosis", Applied Mathematical Sciences, vol. 4, pp. 1065-1072.
10. Mishra, S.; Siddiqui, S.U. and Medhavi, A. (2011), "Blood flow through a composite stenosis in an artery with permeable wall", Applications and Applied Mathematics, vol. 6, pp. 1798 – 1813.
11. Mekheimer, K.S.; Haroun, M.H. and Elkot, M.A. (2011), "Effects of magnetic field, porosity, and wall properties for anisotropically elastic multi-stenosis arteries on blood flow characteristics", Applied Mathematics and Mechanics, vol. 32, pp. 1047-1064.
12. Nanda, S.P. and Mallik, B.B. (2012) "A Non-Newtonian two phase fluid model for blood flow through arteries under stenotic condition", International Journal of Pharmacy and Biological Sciences, vol. 2, pp. 237-247.

13. Shit, G. C.; Roy, M. and Sinha A. (2012), "Mathematical modeling of blood flow through a tapered overlapping stenosed artery with variable viscosity", Applied Bionics and Biomechanics, vol. 11 pp. 185-195.
14. Mathur,P. and Jain, S. (2013) "Mathematical modeling of Non-Newtonian blood flow through artery in the presence of stenosis", Advances in Applied Mathematical Biosciences, vol. 4, pp. 1-12.
15. Hye, M.A. and Paul, M.C. (2015), "A computational study on spiral blood flow in stenosed arteries with and without an upstream curved section", Applied Mathematical Modelling, vol. 39, pp. 4746–4766.
16. Diwakar, C. and Kumar, S. (2016), "Effects of axially symmetric stenosis on the blood flow in an artery having mild stenosis", International Journal of Mathematics Trends and Technology (IJMTT), vol. 35, pp. 163-167.
17. Halder P.; Husain, A.; Zunaid, M. and Samad, A. (2017), "Newtonian and Non-Newtonian pulsatile flows through an artery with stenosis", The Journal of Engineering Research (TJER), vol. 14, pp. 191-205.
18. Ramesh Babu, V. and Savita, T. (2019), "Flow of Jeffrey fluid through an artery with multiple stenosis", International Journal of Engineering Development and Research, vol. 7, pp. 473-477.
19. Liu,Y. and Liu, W. (2019), "Blood flow analysis in tapered stenosed arteries with influence of heat and mass transfer", Journal of Applied Mathematics and Computing, vol. 63, pp. 523-541.
20. Sriyab, S,(2020), "The Effect of Stenotic Geometry and Non-newtonian Property of Blood Flow through Arterial Stenosis", Cardiovascular & Hematological Disorders-Drug Targets, vol. 20, pp.16-30.